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A new normalisation scheme for normal compression and critical state line for soils

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Abstract

Unlike clays, sands may exist in a virgin condition, but at different initial void ratios. Therefore, there is no consensus in the literature as to the definition of the normal compression line for sands. On the basis of experimental evidence, this paper presents a new simple normalisation scheme to unify the compression response of sands. The normalisation scheme incorporates both the initial void ratio and its corresponding equivalent stress on the limiting compression line as normalising parameters. The novelty of the new normalisation scheme is that it is not only consistent with the framework of critical state soil mechanics, but it is also conceptually in line with recent studies on crushability of soils. The validity of the new normalisation scheme is verified against a wide range of experimental data from isotropic and 1-D compression tests on both sands and clays. It is also shown that a unified compression relationship can be obtained for both clays and sands by taking into account the differences in the compression index at high pressure. Similarities between the compression curves and the critical state line for different soils are also examined, and it is shown that the critical state line for clays and sands can be uniquely represented using the proposed normalisation scheme. Areas for further research, where the new normalisation scheme is deemed valuable, are highlighted.

Keywords: Sands, Clays, Triaxial test, Critical state, Compressibility, Normalisation

Introduction

Critical state soil mechanics (CSSM) has long been regarded as the most suitable unified framework to develop predictive constitutive models for geotechnical problems. The framework, initially developed for reconstituted clay, has been widely used to explain many behavioural aspects of other different geo-materials, such as peat [13], sands [2, 11, 22, 39], weak rocks [7, 38] and hard rocks [15, 32].

Despite the fact that many important contributions have been made in recent years, the suitability of the CSSM framework to describe the behaviour of sand is still debatable [22]. The basis of this debate is that, unlike for reconstituted clay, a sand state depends not only on the stress history but also on the mechanism of deposition (e.g. static stress versus vibration). This means that a sand can exist in a freshly deposited state at any arbitrary formation (initial) void ratio with no difference in its stress history. First loading from any of these initial void ratios leads to irrecoverable (plastic) strain during compression. This raises an important question regarding the uniqueness of the normal compression line (NCL) for sands (e.g. [22]). Indeed many attempts have been made

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to define a consistent NCL for sands within the context of CSSM; however, as will be detailed later, this issue has not been resolved satisfactorily.

In an attempt to consistently define the NCL for sands, this paper examines the compression behaviour of sands using experimental results obtained from the literature on different sands subjected to different loading conditions (e.g., isotropic and 1-D compression tests). Based on the experimental results, a new normalisation scheme is proposed to uniquely represent the multitude of compression curves for a sand prepared to different initial densities. Furthermore, the similarities between the isotropic and 1-D compression test results on the one hand and between the NCL and the critical state line (CSL) on the other hand are also examined. Applicability of the proposed normalisation scheme to clays is also verified, and an attempt is made to provide a unified framework to define the NCL for both clays and sands.

Before addressing these issues in detail, it is important to provide a brief background on the compression behaviour of different soils. This is presented in the next few sections.

Background on the compression behaviour of soils

Typical response

Figures 1 and 2 show the idealised compression behaviour of reconstituted clays and sands, respectively. These figures illustrate clearly the intrinsic differences in the compression response between the two soils. The virgin state of a clay always lies on a curve that can be uniquely idealised in terms of the slope λ^* of the compression line, as shown in Fig. 1. However, this simple idealisation cannot be applied to a sand, as it follows more than one compression curve depending on the initial formation density, as shown in Fig. 2.

Definition of the normal compression line for sands

The difference in the compression response between sands and clays, as observed in Figs. 1 and 2, raises a question regarding what comprises the NCL for sands. Two different propositions for the NCL for sands are available in the literature [22]. The first

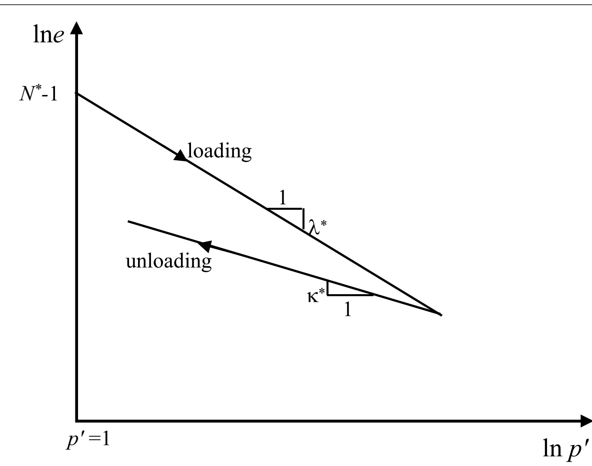

Fig. 1 Idealised compression curves for clays

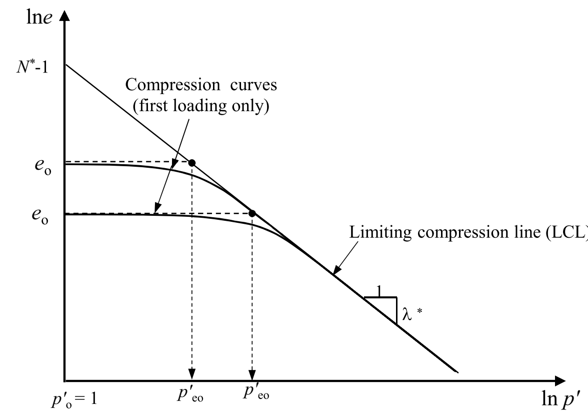

Fig. 2 Idealised compression curves for sands

proposition suggests that, similar to that for clay, the NCL for a sand is unique and it starts only when grain crushing dominates the compression behaviour (e.g. [2, 35]). This is usually referred to as the limiting compression line (LCL), as shown in Fig. 2. On the other hand, following the geological idealisation that any soil experiencing the greatest stress in its history is normally consolidated, the second proposition suggests that the NCL also includes the initial portion of the compression curve before grain crushing becomes dominant. Obviously, this implies existence of an infinite number of normal compression lines for a sand [21, 22].

The implications of both these propositions within the context of CSSM are discussed in Jefferies and Been [22]. They argued that, although the first proposition is consistent with the framework of CSSM, it implies that irrecoverable (plastic) strain occurs only from the evolution of the limiting compression curves (i.e. ignoring the plastic strains that occur during initial loading). On the other hand, the implication of the second proposition is that the infinity of the NCLs cannot be defined within the framework of the CSSM. Jefferies and Been [22] attempted to define the infinity of the NCLs for a sand by taking into consideration the volumetric difference between the current state and the CSL—i.e. the state parameter Ψ [4]. However, the general expressions proposed by Jefferies and Been [22] are rather complicated, limited to low stress levels before grain crushing dominates the compression behaviour, and require a large number of experimental results to validate them. Moreover, the general applicability of their framework to different sands needs to be confirmed through further investigations.

To this end, it is obvious that within the context of the CSSM, neither of the two propositions for the NCL discussed above satisfactorily describes the overall features of the compression behaviour of sand. In general, application of first proposition is limited to the high stress range, whereas the second proposition is limited to the low stress range. A satisfactory definition of the NCL should be able to capture the compression response in both the low and high stress ranges. In addition, it should be consistent for both clays and sands, so that the observed compression behaviour for different soils can be duly idealised. This paper proposes a new normalisation scheme so that compression behaviour of different soils can be consistently described. However, first, the overall importance of normalisation, and the different existing normalisation schemes used in the literature, are discussed in the following section.

Mathematical representation of the normal compression line (NCL)

Within the context of the CSSM, the NCL for a soil is often represented assuming a linear relationship between the void ratio e and logarithm of the effective stress σ' as:

$$e - e_{ref} = -\lambda \, \ell n \, \sigma' \tag{1}$$

where e_{ref} is the reference void ratio (often defined as the void ratio corresponding to an effective stress of unity), λ is the slope of the normal compression line in e-$\ell n\sigma'$ space (or λ^* in $\ell ne - \ell n\sigma'$ space), and σ' is the effective stress (i.e. vertical effective stress σ'_v for 1-D compression and mean effective stress p' for isotropic compression). Although Eq. (1) has been extensively used (e.g. [2, 35]), it is dimensionally inconsistent. Ironically, in their original paper, Roscoe et al. [43] presented the normal compression line in a dimensionless form as:

$$e - e_{ref} = -\lambda \ell n\left(\frac{\sigma'}{\sigma'_o}\right) \tag{2}$$

Roscoe et al. [43] define σ'_o as the effective stress corresponding to e_{ref}. It has been widely argued (e.g. [6, 40]) that, although Eq. (2) provides a dimensionally consistent relationship, its use to represent the compression behaviour is still limited, mainly because:

1. at high pressure most soils show non-linear compression behaviour in an e-lnσ' plot;
2. when $\sigma' > \sigma'_o \exp(e_{\text{ref}}/\lambda)$, Eq. (2) predicts a negative void ratio.

In an attempt to provide a more general and accurate representation of the normal consolidation line for a soil over a wide stress range, Pestana [40] proposed that the normal compression line NCL (or limiting compression line LCL for sand) can be better represented as:

$$\ell n\left(\frac{e}{e_{ref}}\right) = -\lambda^* \ell n\left(\frac{\sigma'}{\sigma'_o}\right) \tag{3}$$

where λ^* is the slope of the normal compression line in $\ell ne - \ell n\sigma'$ plot. It is to be noted that the suffix * is used here to differentiate it from λ obtained from an e-$\ell n\sigma'$ plot.

The above discussions clarify that, although it was realised in the early development of the CSSM [43] that the compression response should be presented in a normalised stress plot [as in Eqs. (2) or (3)], it is surprising to note that no attempt has been made to define a proper normalising parameter for the compression behaviour of soils. In fact it is evident from many later publications that even the importance of the stress normalisation is often ignored (e.g. [2, 35]). This is mainly because most of the research on the CSSM was limited to clay, where the NCL is always assumed to be unique. In this case, the role of stress normalisation is merely to offset the NCL. However, in the case of sand, where a multitude of NCLs exist, proper normalisation is definitely be valuable.

Proposal for a new normalisation scheme

In an attempt to define a unique NCL for all soils, a new normalisation scheme is proposed in this paper. The proposed normalisation scheme consists of the following three elements:

1. Determination of the compression index (λ^*) and normalising stress (σ'_{eo}), which is determined from mapping the initial void ratio (e_o) onto the LCL as shown in Fig. 2. It should be noted that λ (or ¢—slope of the normal compression in $\ell n\nu - \ell n p'$ plot, [6]) can be used instead of λ^*, but steps 2 and 3 below should be modified accordingly.
2. Representation of the compression curves for sand (with different initial densities) in a normalised plot in e/e_o versus $\left(\sigma' - \sigma'_o\right)/\sigma'_{eo}$ space, where e and σ' are the current void ratio and stress, respectively, and σ'_o is the effective stress corresponding to e_o as shown in Fig. 2 (in this paper σ'_o is considered as unity).
3. Representation of the compression curves for different soils in a normalised plot by taking into consideration the differences in λ^* between different soils—i.e. a plot of $(1/\lambda^*)\ell n(e/e_o)$ versus $\left(\sigma' - \sigma'_o\right)/\sigma'_{eo}$.

This simple normalisation scheme will be examined against a large number of experimental results obtained from both sands and clays, but a brief background on the experimental database is presented first.

Details of the experimental database

Sand samples from different origins covering a wide range of initial void ratio (e_o) and compression parameters λ^* and N^* were examined as summarised in Tables 1, 2 and 3; where λ^* and N^* are the slope and intercept of the LCC in ln$e$-lnp' plot as shown in Fig. 2. (It should be noted that the derivation of these parameters for sands requires tests to be performed to high stress). The maximum stress values σ'_{max}: p'_{max} for isotropic tests and $\sigma'_{v\,max}$ for 1-D tests) reached in each test are also reported in Tables 1, 2 and 3. Results obtained from both isotropic and 1-D compression tests were used. The experimental database on the compression response of different clays and the CSL for different soils are presented in Tables 4 and 5, respectively. (Note that most of the test results were collected from the published literature, and only a few tests were performed by the authors). Additional details on the soil properties can be found in the relevant references listed in Tables 1, 2, 3, 4 and 5, and hence are not discussed here.

Experimental validation of the new normalisation scheme for different sands

Application of the new normalisation scheme to the experimental data presented above led to the results shown in Figs. 3, 4 and 5 in the normalised $(1/\lambda^*)\ell n\ (e/e_o)$ versus $\left(\sigma' - \sigma'_o\right)/\sigma'_{eo}$ space. It should be noted that the samples were tested at different initial void ratios as reported in Tables 3, 4 and 5. Figure 3 presents results from 31 individual isotropic compression tests on 15 different types of sands. Similarly, results obtained from 44 individual anisotropic compression tests on 16 different types of sands are presented in Fig. 4 and additional results obtained from 1-D compression tests [8] are shown in Fig. 5. [Each soil in Figs. 3, 4 and 5 was given a different symbol and the soil name (soil ID) reported in the legend of each figure is in accordance with the soil ID

Table 1 Database of isotropic compression tests for different sands

S. no	Soil	Soil ID	p'_{max} (MPa)	e_o	N^*	λ^*	References
1	Aio	AO	10.00	0.760	17.09	0.28	Hyodo et al. [19]
			10.12	0.652			
2	Chattachooche river	CHA	63.01	0.723	35.48	0.34	Vesic and Clough [48]
3	Chiibashi	CHI	41.00	1.044	45.92	0.35	Kato et al. [24]
4	Dogs Bay	DB	7.98	1.709	62.68	0.37	Coop [11]
			7.26	1.633			
			7.63	1.444			
5	North Rankin	NR	65.56	1.120	85.75	0.42	Huang [17]
6	Goodwyn	GW	10.19	1.740	72.97	0.39	Sharma [45]
7	Ham river	HR	8.85	0.799	55.02	0.36	Sharma [45]
			47.71	0.692			
			18.47	0.689			
8	Ledge point	LP	10.00	1.074	20.82	0.25	Sharma [45]
			14.20	0.974			
			14.30	0.901			
9	Masado	MA	41.65	0.902	4.89	0.15	Kato et al. [24]
10	Rottnest	RT	56.08	0.940	65.17	0.38	Ismail [20]
11	Sacramento river	SAR	13.97	0.870	20.02	0.37	Lee and Seed [27]
			12.88	0.778			
			14.13	0.711			
			14.42	0.608			
12	Shirasu	SHI	40.43	1.289	32.07	0.31	Hyodo et al. [19]
			42.61	1.143			
			3.05	0.922			
13	Silica 1.4–1.8	SI1	40.11	0.647	34.52	0.43	Kato et al. [24]
14	Silica 1.8–2.0	SI2	39.45	0.620	36.74	0.34	Kato et al. [24]
15	Toyoura	TO	3.97	0.908	24.49	0.37	Miura et al. [33]
			18.75	0.840			
			48.68	0.770			
			3.93	0.610			
			53.70	0.590			

p'_{max} is the maximum mean effective stress applied during consolidation

listed in the corresponding table; soil ID in Figs. 3, 4 and 5 corresponds to Tables 1, 2 and 3, respectively].

It is encouraging to note from Fig. 3 that a reasonably unique isotropic normal compression line exists for different sands covering a wide range of initial void ratios and compression characteristics. A similar pattern can be observed from the anisotropic compression test results presented in Figs. 4 and 5.

Direct comparison of the results from the isotropic and anisotropic compression tests results presented in Figs. 3, 4 and 5 is shown in Fig. 6 in terms of $(1/\lambda^*)\ell n\ (e/e_o)$ versus $\left(\sigma' - \sigma'_o\right)/\sigma'_{eo}$, where σ' is the effective stress and is equal to σ'_v for 1-D compression tests and p' for isotropic compression tests. It can be concluded from Figs. 3, 4, 5 and 6 that a reasonably unique normalised NCL exists for all sands, irrespective of their initial void ratio, origin (mineralogy) and the type of test. In other words, the differences in the compression response between different sands tested under different loading conditions lie in the differences in the compression parameters λ^* and N^*.

Table 2 Database of 1-D compression tests for different sands

S. no	Soil	Soil ID	$\sigma'_{v\,max}$ (MPa)	e_o	N^*	λ^*	References
1	Cambria sand	CM	816.00	0.696	93.77	0.432	Yamamuro et al. [49]
			805.14	0.599			
			838.90	0.549			
2	Glauconite sand	GL	28.92	0.835	158.90	0.463	As reported in Pestana [40]
			32.33	0.765			
			44.67	0.632			
			38.80	0.614			
3	Ground dolomite-1	GD1	142.94	0.984	64.99	0.400	Roberts [42] (as reported in [40])
4	Ground dolomite-2	GD2	70.66	0.792	76.81	0.432	
5	Ground feldspar	GF	144.56	1.104	61.39	0.379	Roberts [42] (as reported in [40])
			141.55	1.079			
			112.20	0.926			
6	Ground quartz-1	GQ1	141.13	1.052	54.25	0.372	Roberts [42] (as reported in [40])
7	Ground quartz-2	GQ2	138.22	0.829	62.17	0.394	
8	Gypsum sand	GY	790.00	0.774	172.24	0.535	Yamamuro et al. [49]
			802.00	0.715			
			790.84	0.674			
9	Hawaiian sand	HS	138.29	0.833	80.66	0.397	Roberts [42] (as reported in [40])
			174.72	0.726			
10	Hokksund sand	HK	24.13	0.754	68.66	0.391	As reported in Pestana [40]
			24.13	0.715			
			49.40	0.547			
			49.40	0.530			
11	Ottawa	OT	138.04	0.821	245.99	0.465	DeSouza [14] (as reported in [40])
			141.25	0.763			
			136.91	0.673			
12	Quiu sand	QU	23.21	0.999	25.04	3.220	As reported in Pestana [40]
			23.47	0.903			
			48.44	0.822			
13	Quartz sand	QT	808.71	0.899	79.39	0.383	Yamamuro et al. [49]
			798.16	0.679			
14	Sangamon river	SAN	19.22	0.800	32.61	0.288	DeSouza [14] (as reported in [40])
			19.26	0.758			
			19.14	0.715			
			19.17	0.656			
			19.19	0.630			
15	Ticino sand	TI	48.08	0.799	69.21	0.392	Hendron [16] (as reported in [40])
			47.59	0.676			
			49.15	0.602			
16	Wabash river sand	WR	19.09	0.675	36.83	0.329	Hendron [16] (as reported in [40])
			19.09	0.630			
			19.27	0.572			
			19.14	0.545			
			19.09	0.502			

$\sigma'_{v\,max}$ is the maximum vertical effective stress applied during consolidation

Compression response for clays

It is now interesting to examine the compression response of clays in a manner similar to that discussed above for sands; the aim is to investigate whether a unified trend will

Table 3 Database of 1-D compression tests for different sands (results from [8])

S. no	Soil	Soil ID	D_{60} (mm)	$\sigma'_{v\,max}$ (MPa)	e_o	N^*	λ^*
1	Mono-quartz-rich-1	MQ-1	0.33	49.41	0.810	152.12	0.45
2	Mono-quartz-rich-2	MQ-2	2.30	48.66	0.733	144.76	0.50
3	Mono-quartz-rich-3	MQ-3	1.51	47.88	0.744	177.29	0.50
4	Mono-quartz-rich-4	MQ-4	0.85	50.28	0.758	145.78	0.47
5	Mono-quartz-rich-5	MQ-5	0.71	50.66	0.735	160.07	0.47
6	Mono-quartz-rich-6	MQ-6	0.33	50.77	0.751	138.98	0.44
7	Mono-quartz-rich-7	MQ-7	0.19	49.23	0.734	23.43	0.26
8	Mono-quartz-rich-8	MQ-8	0.10	48.58	0.746	64.43	0.36
9	Mono-quartz-rich-9	MQ-9	0.19	50.13	0.893	83.88	0.38
10	Mono-quartz-rich-10	MQ-10	0.38	49.28	0.869	122.57	0.44
11	Mono-quartz-rich-11	MQ-11	0.71	49.81	0.747	100.59	0.46
12	Mono-quartz-rich-12	MQ-12	0.1	53.87	0.923	48.24	0.33
13	Mono-quartz-rich-13	MQ-13	0.1	52.48	0.751	25.84	0.27
14	Mono-quartz-rich (SR)	MQR	0.17	49.51	0.906	62.25	0.34
15	Mono-quartz-rich (SA)	MQA	0.19	48.32	0.891	82.65	0.38
16	Quartz (mixed-1)	QM-1	–	47.04	0.808	109.02	0.43
17	Quartz (mixed-2)	QM-2	0.33	48.68	0.809	115.84	0.43
18	Quartz (mixed-3)	QM-3	–	40.86	0.696	29.73	0.34
19	Lithic-1	LI-1	1.46	50.60	0.885	143.87	0.48
20	Lithic-2	LI-2	0.72	52.31	0.869	176.87	0.49
21	Lithic-3	LI-3	0.37	48.05	0.866	141.43	0.45
22	Lithic-4	LI-4	0.17	50.60	0.882	108.17	0.42
23	Lithic-5	LI-5	0.17	49.81	0.870	89.01	0.40
24	Lithic-6	LI-6	0.37	47.52	0.891	130.99	0.45
25	Lithic-7	LI-7	0.72	48.86	0.776	103.34	0.44
26	Lithic-8	LI-8	1.46	48.56	0.758	107.77	0.46
27	Carbonate-1	CA-1	1.48	40.60	1.190	139.41	0.47
28	Carbonate-2	CA-2	0.75	47.99	1.165	130.16	0.46
29	Carbonate-3	CA-3	0.38	49.23	1.178	140.63	0.46
30	Carbonate-4	CA-4	0.19	47.12	1.171	130.44	0.44
31	Carbonate-5	CA-5	0.19	50.00	0.870	101.68	0.42
32	Carbonate-6	CA-6	0.38	49.87	0.867	138.28	0.45
33	Carbonate-7	CA-7	0.75	49.00	1.037	167.72	0.48
34	Carbonate-8	CA-8	1.48	1.36	1.200	189.07	0.50
35	Carbonate-9	CA-9	0.75	49.16	1.409	265.89	0.54
36	Carbonate-10	CA-10	2.8	50.93	1.652	349.29	0.56
37	Carbonate-11	CA-11	2.8	49.65	1.402	318.56	0.56
38	Carbonate-12	CA-12	0.75	51.34	1.157	179.07	0.49
39	Carbonate-13	CA-13	0.75	50.45	1.032	206.75	0.50
40	Carbonate-14	CA-14	0.19	50.45	1.178	156.49	0.46
41	Carbonate-15	CA-15	0.19	50.93	0.871	141.17	0.45
41	Chirt-rich	CH	1.67	49.31	0.717	54.82	0.34

SR sub rounded, *SA* sub angular

emerge for both sands and clays whose compression behaviour has always been treated separately in the literature. To this end, a large number of experimental results obtained from different clays were also examined, using the database listed in Table 4 for clays.

Table 4 Isotropic and 1-D compression test results for different clays

S. no	Soil	Soil ID	p'_{max} (MPa)	e_o	N^*	λ^*	References
Isotropic compression tests							
1	Black cotton	BC	0.81	4.219	5.22	0.198	Nagaraj and Srinivasa Murthy [37]
2	Boston blue	BB-1	0.79	3.060	4.06	0.180	Taylor [47]
3	Chicago	CG	1.61	4.033	5.03	0.227	Taylor [47]
4	Drammen (plastic)	DP	0.81	3.666	4.67	0.223	Butterfield [6]
5	Drammen (lean)	DL	0.69	1.655	2.65	0.135	Butterfield [6]
6	Kaolin	KA-1	0.69	4.400	5.40	0.193	Amerasinghe [1]
7	Litle belt	LB	0.78	5.014	6.01	0.186	Hvorslev [18]
8	Newfoundland	NS	1.66	3.064	4.06	0.193	Taylor [47]
9	Newfoundland	NP	0.80	61.614	62.61	0.418	Taylor [47]
10	Red soil	RC	0.78	1.557	2.56	0.150	Nagaraj and Srinivasa Murthy [37]
11	residual soil	RS	0.78	1.899	2.90	0.133	Leonards and Ramiah [28]
12	Sail soil	SI	0.82	6.295	7.29	0.182	Nagaraj and Srinivasa Murthy [37]
13	Soft clay	SO	0.79	3.281	4.28	0.178	Nagaraj and Srinivasa Murthy [37]
14	silty clay	SC-1	0.80	2.459	3.46	0.156	Nagaraj and Srinivasa Murthy [37]
15	silty clay	SC-2	0.80	1.152	2.15	0.125	Jennings and Burland [23]
16	Silty clay	SC-3	0.80	1.750	2.75	0.128	Jennings and Burland [23]
17	Vienna	VI	0.79	1.685	2.69	0.146	Hvorslev [18]
18	Weald	WD	0.56	4.098	5.10	0.373	Roscoe et al. [43]
19	Whangamarino	WG	0.79	4.851	5.85	0.152	Nagaraj and Srinivasa Murthy [37]
1-D compression tests							
1	Argile plastique	AP	2.01	4.906	5.91	0.227	Burland [5]
2	Boston blue	BB-2	100.00	2.224	3.22	0.175	Pestana [40]
3	Bothkennear	BT	0.99	3.103	4.10	0.196	Smith et al. [46]
4	Detroit	DT	27.17	1.738	2.74	0.190	Lambe and Whitman [26]
5	Hiroshima	HI	1.26	3.852	4.85	0.211	Moriwaki et al. [34]
6	Horten	HO	1.04	1.144	2.14	0.143	Lambe and Whitman [26]
7	Kaolin	KA-2	0.51	5.373	6.37	0.199	Nadarajah [36]
8	Kleinbelt Ton	KT	1.06	5.373	6.37	0.204	Burland [5]
9	Kurashiki	KU	1.39	2.795	3.80	0.191	Moriwaki et al. [34]
10	London	LC	4.66	3.276	4.28	0.222	Burland [5]
11	Lower cromer	LT	1.01	0.974	1.97	0.151	Burland [5]
12	Magnus	MG	3.99	1.789	2.79	0.193	Burland [5]
13	Maizuru	MZ	1.29	4.283	5.28	0.221	Moriwaki et al. [34]
14	Mexico city	MC	2.84	42.606	43.61	0.362	Mesri et al. [31]
15	Shellhaven	SL	0.61	3.609	4.61	0.187	Lambe and Whitman [26]
16	Tilbury	TI	0.16	4.608	5.61	0.205	Lambe and Whitman [26]
17	Wiener Tegel	WT	1.04	1.761	2.76	0.162	Burland [5]

Figure 7 shows the normalised compression lines for all the clays reported in Table 4. Indeed, the new normalisation scheme reasonably unifies the normal compression lines for the wide range of clays examined.

Unified compression curves for sands and clays

It is shown in the previous sections that the compression behaviour of different sands and clays (separately) can be uniquely represented using the new normalisation scheme. An obvious step forward thus would be to directly compare the compression response of

Table 5 Database on critical state line for different soils

S. no	Soil	Soil ID	p'_{max} (MPa)	e_o	Γ^*	λ^*	References
Clays							
1	Kaolin	KAc	0.70	3.149	4.15	0.159	Schofield and Wroth [44]
2	Klein Belt	KBc	0.70	4.378	5.38	0.180	Schofield and Wroth [44]
3	London	LNc	0.70	2.014	3.01	0.158	Schofield and Wroth [44]
4	Wiener	WIc	0.70	1.640	2.64	0.168	Schofield and Wroth [44]
5	Weald	WEc	0.60	3.410	4.41	0.370	Roscoe et al. [43]
Sands							
1	Chattachooche river	CRc	19.12	0.944	23.82	0.440	Muir Wood [35]
2	Chiibashi	CHc	10.04	1.319	12.45	0.350	Kato et al. [24]
3	Dogs Bay	DBc	6.94	1.733	13.97	0.371	Coop [11]
4	Erksak	ERc	2.81	0.772	10.55	0.350	Been et al. [3]
5	Ham river	HRc	56.05	0.900	14.24	0.360	Coop and Lee [10]
6	Leighton Buzzard	LBc	4.79	0.969	4.72	0.200	Been et al. [3]
7	Masado	MSc	61.68	0.729	2.49	0.148	Kato et al. [24]
8	Massey tunnel	MTc	1.00	1.094	7.23	0.298	Konrad [25]
9	Quebec	QBc	1.12	0.750	9.15	0.389	Konrad [25]
10	Sacramento river	SRc	18.55	0.951	16.25	0.387	Riemer et al. [41], Lee and Seed [27]
11	Silica 1.4–1.7	SIc-1	75.95	0.841	17.21	0.433	Kato et al. [24]
12	Silica 1.8–2.0	SIc-2	34.94	0.759	9.21	0.340	Kato et al. [24]
13	Ticino	TIc	1.11	0.910	9.66	0.365	Konrad [25]
14	Toyoura	TOc	35.49	0.926	17.98	0.376	Miura et al. [33]
15	Tung-Chung	TCc	0.70	0.870	4.62	0.253	Li and Wang [29]
Other soils							
1	Johnstone	JTc	50.00	0.580	8.80	0.335	Novello and Johnston [38]
2	Ohya tuff	OTc	50.00	0.860	4.56	0.197	Novello and Johnston [38]

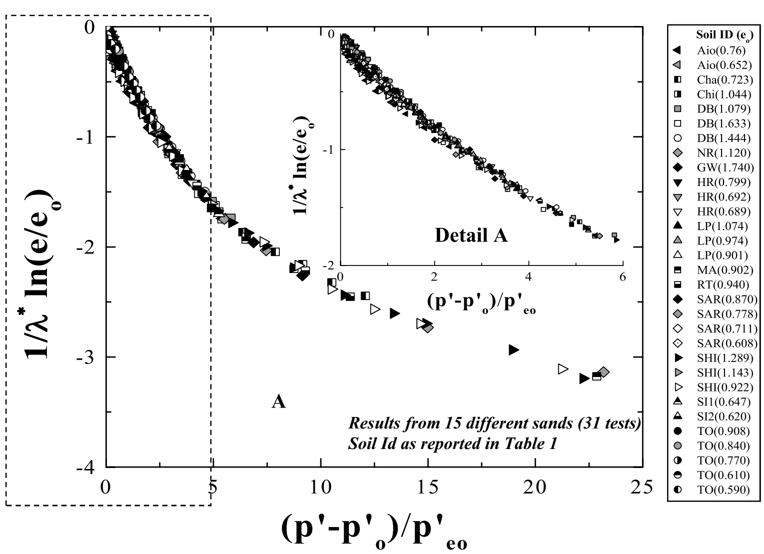

Fig. 3 Normalised compression response for different sands (results from isotropic tests only)

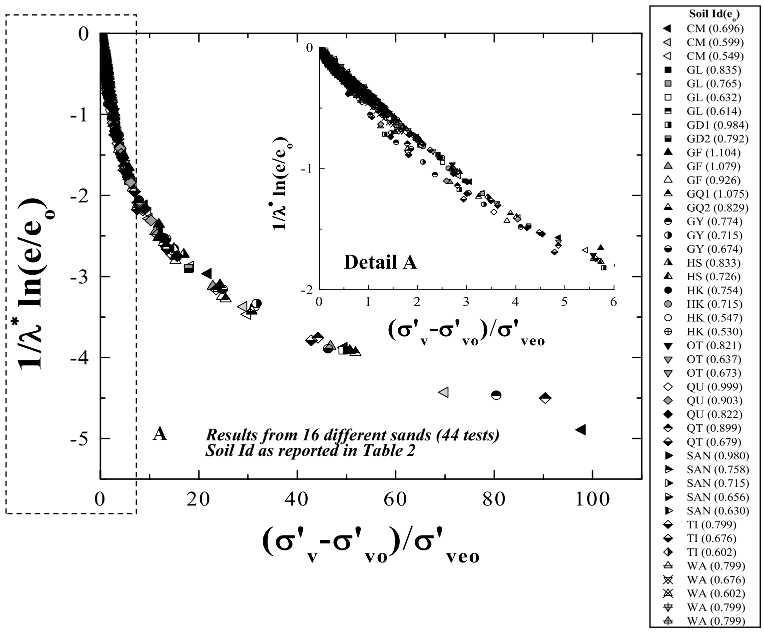

Fig. 4 Normalised compression response for different sands (results from 1-D compression tests: Table 2)

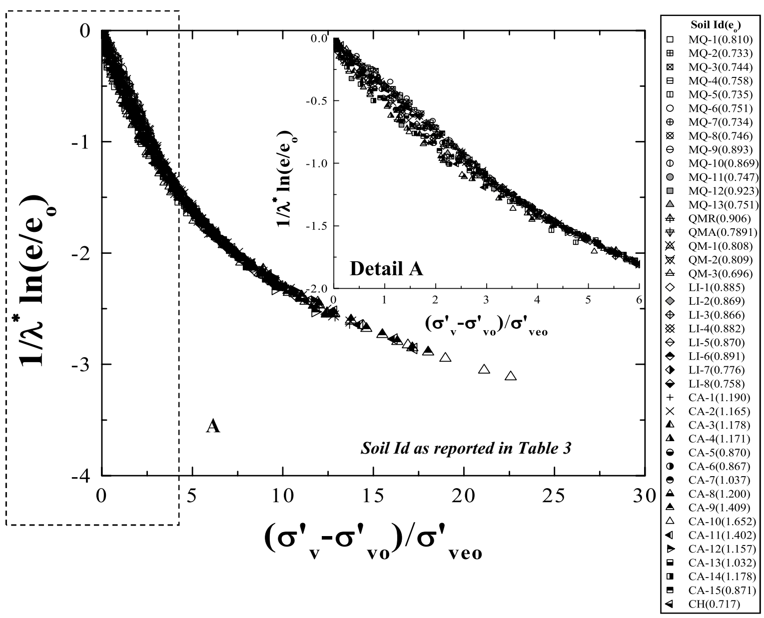

Fig. 5 Normalised compression response for different sands (results from 1-D compression tests: Table 3)

sands and clays so that a unified compression relationship could be developed for both soils.

The experimental results presented above for both sands and clays are replotted in Fig. 8. Although there is some scatter in the results at low stress levels, the overall

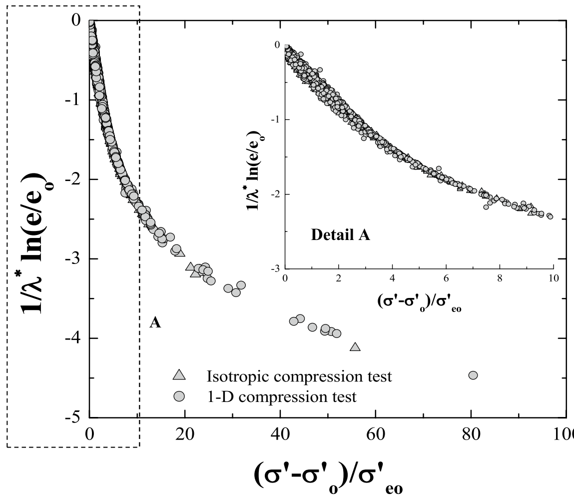

Fig. 6 Normalised compression response of different sands from isotropic and 1-D compression tests

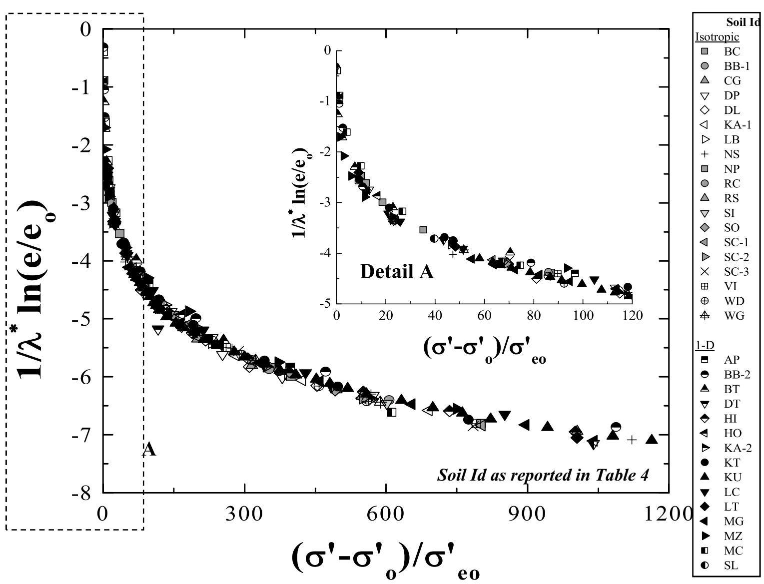

Fig. 7 Normalised compression response for different clays (results from both isotropic and 1-D compression tests)

compression behaviour of all tested sands and clays lie within a narrow range and can be represented uniquely with reasonable accuracy.

The compression line versus the critical state line

The similarity between the normal compression line (NCL) and critical state line (CSL) for any particular soil is widely reported in the literature. In particular for clay, it is well accepted that the CSL is parallel to the normal consolidation line in the $\ell ne - \ell np'$ space (e.g. [6, 40]). However, it is difficult to obtain a similar simple relationship for different sands. This is mainly because, unlike for the clays, the CSL for sands is non-linear in the

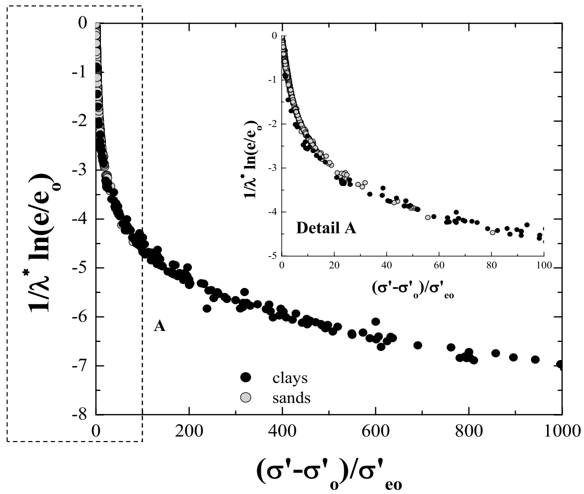

Fig. 8 Unified relationship for normalised compression response for clays and sands

$\ell ne - \ell np'$ space ([33, 40, 41]) and can be represented by a schematic diagram as shown in Fig. 9.

It is envisaged that applying the proposed normalisation scheme to the CSL would be valuable for modelling purposes. Normalising the CSL involves plotting $(1/\lambda^*)\ell n(e_c/e_{oc})$ versus $\left(p'_c - p'_{oc}\right)/p'_{eoc}$, where the various parameters used are the same as for the NCL normalisation, except that they refer to the CSL instead of the NCL, as indicated by the 'c' subscript (see Fig. 9).

The normalisation is examined in Fig. 10, which shows the CSL for different soils varying from clay to soft rocks (reported in Table 5) in a normalised plot. Again, the CSLs for the wide range of soils examined fall within a narrow range and can be represented with reasonable accuracy using an average line passing through the data points.

A direct comparison of the normalised CSL and NCL for the examined soils is presented in Fig. 11 using both the CSL and NCL normalising parameters. It can be clearly observed that all the results fall within a narrow range and can be represented uniquely with reasonable accuracy using the appropriate normalisation for the two lines. This

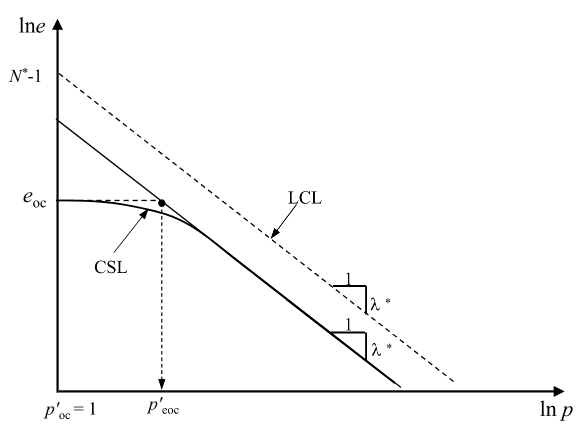

Fig. 9 Idealised critical state line for sands

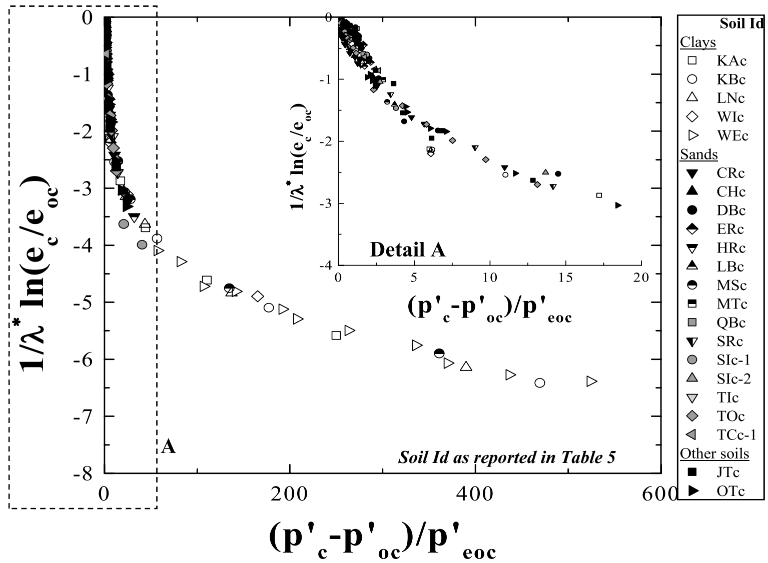

Fig. 10 Critical state line for different soils

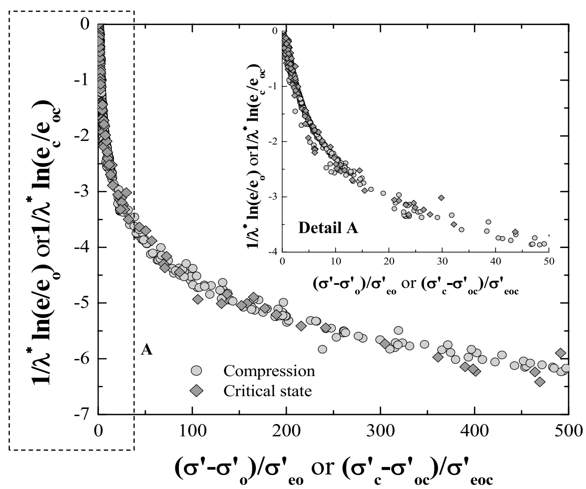

Fig. 11 Compression versus critical state line for different soils in normalised plot

finding is quite important, as it suggests that the general shape of the CSL and NCL is similar for all types of soils in this normalised space, and it can be represented using a similar mathematical form; the difference lies only in the value of the normalising parameters i.e. the initial void ratio e_o for the normal compression line and e_c for the critical state line.

Discussion and areas for further research

The new normalisation scheme presented above provides a unified framework to describe the compression behaviour of different soils and can be readily implemented for modelling purposes. The method takes into account the initial void ratio e_o and the corresponding equivalent stress σ'_{eo} (σ'_{eo} corresponds to p'_{eo} for isotropic tests and σ'_{veo} for

1-D tests) as normalising parameters. In particular, the novelty of the proposed normalisation scheme lies in adopting σ'_{eo} as a stress normalising parameter. This stress normalisation is not only consistent with the framework of CSSM, but also can be considered conceptually similar to some of the recent studies on crushability of soils (e.g. [30]).

In addition, it should be noted that e/e_o is a measure of the volumetric strain, which makes the new normalisation scheme more versatile for practical applications and also to develop thermodynamically consistent constitutive models (e.g. [9]), where the volumetric strain (not the void ratio) can only be used as a state variable.

It is a well-known fact that the presence of "structure", either in the form of a fabric or bonding [5], plays an important role in controlling the compression behaviour of natural soils. The results presented in this paper were obtained from reconstituted samples and the new normalisation scheme has been validated only against them. The applicability of the proposed method to structured soil (natural or artificially structured soils) needs further study. However, since the presence of a structure causes the compression curves to move towards the right in the $\ell n e - \ell n \sigma'$ space (e.g. [12]), it can be imagined that the LCL of structured soil needs to be considered in defining the value of σ'_{eo}. The proposal suggested by Cuccovillo and Coop [12] in defining the compression boundary for structured material could be valuable in this regard.

A unique representation of the NCL and CSL discussed above for different soils is believed to be valuable for developing unified constitutive models within the framework of CSSM and also for proposing unified empirical relationships for different soils. In the case of constitutive modelling, the results presented above can be used to define the yield surface and the hardening rule for different soils. It is also believed to be worth examining the applicability of the new normalisation scheme to develop a unified state boundary surface for different soils.

The proposed normalisation scheme could be used to develop a unified empirical relationship for the qualities that depends on the sand state. For example, since the new normalisation scheme uniquely correlates the void ratio and effective stress for different soils, it may be used to unify the expression used for the small-strain shear modulus of different soils.

Summary and conclusions

In this paper, a new normalisation scheme is proposed to uniquely represent the compression curves for different soils subjected to different compression loading conditions (1-D or isotropic compression tests). It has been validated against a large number of experimental results extracted from the literature. It is shown that using the new normalisation scheme the compression response of a sand of different initial void ratios can be uniquely represented. Also, by taking into consideration the differences in the compression index between different soils, it is shown that a unique compression curve can be obtained in the normalised plot. The similarities between the compression curves and critical state line are also validated. The new normalisation scheme is consistent with the framework of the CSSM and crushability of soils. The potential applications of the new normalisation scheme to develop unified constitutive and empirical relationship for different soils are also discussed.

Authors' contributions
SSS collected and analysed the data and wrote most of the sections of the paper. MAI verified the proposed normalisations scheme, validated the graphs, and wrote some sections of the paper. Both authors read and approved the final manuscript.

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Competing interests
The authors declare that they have no competing interests.

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